

ANALYSIS OF THE Σ_Q BARYONS IN THE NUCLEAR MATTER WITH THE QCD SUM RULES

Zhi-Gang Wang¹

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we extend our previous work to study the Σ -type heavy baryons Σ_c and Σ_b in the nuclear matter using the QCD sum rules, and obtain three coupled QCD sum rules for the masses $M_{\Sigma_Q}^*$, vector self-energies Σ_v and pole residues $\lambda_{\Sigma_Q}^*$ in the nuclear matter. Then we take into account the effects of the unequal pole residues from different spinor structures, and normalize the masses from the QCD sum rules in the vacuum to the experimental data, and obtain the mass-shifts $\delta M_{\Sigma_c} = -123$ MeV and $\delta M_{\Sigma_b} = -375$ MeV in the nuclear matter.

PACS numbers: 12.38.Lg; 14.20.Lq; 14.20.Mr

Key Words: Nuclear matter, QCD sum rules

1 Introduction

The QCD sum rules is a powerful theoretical tool in studying the ground state hadrons both in the vacuum and in the nuclear matter, and has given many successful descriptions of the hadron properties, such as the masses, decay constants, form-factors, hadron coupling constants, etc [1]. The properties of the light-flavor mesons and baryons in the nuclear matter have been studied extensively with the QCD sum rules [2, 3, 4], while the existing works on the heavy quarkonia and heavy baryons in the nuclear matter focus on the J/ψ , η_c , D , B , D_0 , B_0 , D^* , D_1 , Λ_c and Λ_b [5, 6, 7]. The in-medium mass modifications of the $Q\bar{q}$ and Qqq hadrons differ greatly from the corresponding ones of the $q\bar{q}$ and qqq hadrons due to the appearance of the heavy quark, the full propagators of the heavy quarks in the nuclear matter undergo much slight modifications compared with that of the light quarks. The upcoming FAIR (facility for antiproton and ion research) project at GSI (heavy ion research lab) provides the opportunity to extend the experimental studies of the in-medium properties of the mesons and baryons into the charm sector [8, 9], we can study the heavy hadrons in nuclear matter with the QCD sum rules and make predictions to be confronted with the experimental data of the CBM (compressed baryonic matter) and PANDA collaborations in the future.

The scattering amplitude of one-gluon exchange can be rephrased into an antisymmetric antitriplet $\bar{\mathbf{3}}_c$ and an symmetric sextet $\mathbf{6}_c$ in the color-space. The attractive interaction in the antisymmetric antitriplet favors the formation of the diquark states in the color antitriplet $\bar{\mathbf{3}}_c$, the most stable diquark states, the Λ -type diquark states, maybe exist in the color antitriplet $\bar{\mathbf{3}}_c$, flavor antitriplet $\bar{\mathbf{3}}_f$ and spin singlet $\mathbf{1}_s$ channels due to Fermi-Dirac statistics [10], and the next stable diquark states, the Σ -type diquark states, maybe exist in the color antitriplet $\bar{\mathbf{3}}_c$, flavor sextet $\mathbf{6}_f$ and spin triplet $\mathbf{3}_s$ channels [11]. In the heavy quark limit, the heavy baryons can be classified as the Λ -type or Σ -type baryons according to the spin structures of the two light quarks [12]. In Ref.[7], we study the Λ -type heavy baryons Λ_c and Λ_b in the nuclear matter using the QCD sum rules, and obtain the in-medium positive mass-shifts $\delta M_{\Lambda_c} = 51$ MeV and $\delta M_{\Lambda_b} = 60$ MeV, respectively. In this article, we extend our previous work to study the properties of the Σ -type heavy baryons Σ_c and Σ_b in the nuclear matter using the QCD sum rules.

The article is arranged as follows: we study the heavy baryons Σ_c and Σ_b in the nuclear matter with the QCD sum rules in Sec.2; in Sec.3, we present the numerical results and discussions; and Sec.4 is reserved for our conclusions.

¹E-mail, wangzgyiti@yahoo.com.cn.

2 The in-medium Σ_Q baryons with QCD sum rules

We study the Σ_c and Σ_b baryons in the nuclear matter with the following two-point correlation functions $\Pi(q)$,

$$\begin{aligned}\Pi(q) &= i \int d^4x e^{iq \cdot x} \langle \Psi_0 | T \{ J(x) \bar{J}(0) \} | \Psi_0 \rangle, \\ J(x) &= \epsilon^{ijk} u_i^t(x) C \gamma_\mu d_j(x) \gamma^\mu \gamma_5 Q_k(x),\end{aligned}\quad (1)$$

where the i, j, k are color indexes, $Q = c, b$, the C is the charge conjugation matrix, and the $|\Psi_0\rangle$ is the nuclear matter ground state. The correlation functions $\Pi(q)$ can be decomposed as

$$\Pi(q) = \Pi_s(q^2, q \cdot u) + \Pi_q(q^2, q \cdot u) \not{q} + \Pi_u(q^2, q \cdot u) \not{u}, \quad (2)$$

according to Lorentz covariance, parity and time reversal invariance [2, 3]. In the limit $u_\mu = (1, 0)$, the component $\Pi_i(q^2, q \cdot u)$ reduces to $\Pi_i(q_0, \vec{q})$, where $i = s, q, u$.

We insert a complete set of intermediate heavy baryon states with the same quantum numbers as the current operators $J(x)$ into the correlation functions $\Pi(p)$ to obtain the hadronic representation [1], then isolate the ground state Σ_Q baryon contributions, and resort to the dispersion relation to rephrase the three components of the correlation functions $\Pi_i(q_0, \vec{q})$ in the following form:

$$\Pi_i(q_0, \vec{q}) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta \Pi_i(\omega, \vec{q})}{\omega - q_0}, \quad (3)$$

where

$$\begin{aligned}\Delta \Pi_s(\omega, \vec{q}) &= -2\pi i \frac{\lambda_{\Sigma_Q}^{*2} M_{\Sigma_Q}^*}{2E_q^*} [\delta(\omega - E_q) - \delta(\omega - \bar{E}_q)], \\ \Delta \Pi_q(\omega, \vec{q}) &= -2\pi i \frac{\lambda_{\Sigma_Q}^{*2}}{2E_q^*} [\delta(\omega - E_q) - \delta(\omega - \bar{E}_q)], \\ \Delta \Pi_u(\omega, \vec{q}) &= +2\pi i \frac{\lambda_{\Sigma_Q}^{*2} \Sigma_v}{2E_q^*} [\delta(\omega - E_q) - \delta(\omega - \bar{E}_q)],\end{aligned}\quad (4)$$

$E_q^* = \sqrt{M_{\Sigma_Q}^{*2} + \vec{q}^2}$, $E_q = \Sigma_v + E_q^*$, $\bar{E}_q = \Sigma_v - E_q^*$, the $M_{\Sigma_Q}^*$, Σ_v and $\lambda_{\Sigma_Q}^*$ are the masses, vector self-energies and pole residues of the Σ_Q baryons respectively in the nuclear matter.

We carry out the operator product expansion in the nuclear matter at the large space-like region $q^2 \ll 0$, and obtain the spectral densities at the level of quark-gluon degrees of freedom, then take the limit $u_\mu = (1, 0)$, and write the three components $\Pi_i(q_0, \vec{q})$ in the following form [2, 3]:

$$\Pi_i(q_0, \vec{q}) = \sum_n C_n^i(q_0, \vec{q}) \langle O_n \rangle_{\rho_N}, \quad (5)$$

where the $C_n^i(q_0, \vec{q})$ are the Wilson coefficients, the $\langle O_n \rangle_{\rho_N}$, which are defined as $\langle \Psi_0 | O_n | \Psi_0 \rangle$, are the condensates in the nuclear matter and can be decomposed as $\langle \mathcal{O} \rangle + \rho_N \langle \mathcal{O} \rangle_N$ at the low nuclear density, the $\langle \mathcal{O} \rangle$ and $\langle \mathcal{O} \rangle_N$ denote the vacuum condensates and the nuclear matter induced condensates, respectively. The imaginary parts of the QCD spectral densities can be obtained through the formula $\Delta \Pi_i(\omega, \vec{q}) = \lim_{\epsilon \rightarrow 0} [\Pi_i(\omega + i\epsilon, \vec{q}) - \Pi_i(\omega - i\epsilon, \vec{q})]$.

Finally, we match the hadronic spectral densities with the QCD spectral densities, and multiply both sides with the weight function $(\omega - \bar{E}_q) e^{-\frac{\omega^2}{M^2}}$, perform the integral $\int_{-\omega_0}^{\omega_0} d\omega$,

$$\int_{-\omega_0}^{\omega_0} d\omega \Delta \Pi_i(\omega, \vec{q}) (\omega - \bar{E}_q) e^{-\frac{\omega^2}{M^2}}, \quad (6)$$

to exclude the negative-energy pole contributions, and obtain the following three QCD sum rules:

$$\begin{aligned}
\lambda_{\Sigma_Q}^{*2} e^{-\frac{E_Q^2}{M^2}} &= \int_{m_Q^2}^{s_0^*} ds \int_{x_i}^1 dx \left\{ \frac{x(1-x)^3(s - \hat{E}_Q^2)(5s - 3\hat{E}_Q^2)}{32\pi^4} - \frac{2x(1-x)\langle q^\dagger i D_0 q \rangle_{\rho_N}}{3\pi^2} \right. \\
&\quad \left[2 + (\tilde{m}_Q^2 - 2s)\delta(s - \hat{E}_Q^2) \right] + \frac{1}{96\pi^2} \langle \frac{\alpha_s GG}{\pi} \rangle_{\rho_N} \left[(4 - 5x) + (1-x)\tilde{m}_Q^2\delta(s - \hat{E}_Q^2) \right] \\
&\quad - \frac{m_Q^2}{144\pi^2} \langle \frac{\alpha_s GG}{\pi} \rangle_{\rho_N} \frac{(1-x)^3}{x^2} \left(1 + \frac{\tilde{m}_Q^2}{2M^2} \right) \delta(s - \hat{E}_Q^2) + \bar{E}_q \left[\frac{x(1-x)\langle q^\dagger q \rangle_{\rho_N}}{2\pi^2} \right. \\
&\quad - \frac{x\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{6\pi^2} \delta(s - \hat{E}_Q^2) + \frac{x(1-x)\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{12\pi^2} \left(1 + \frac{2s}{M^2} \right) \delta(s - \hat{E}_Q^2) \\
&\quad - \frac{x\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N}}{3\pi^2} \delta(s - \hat{E}_Q^2) - \frac{x(1-x)\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N}}{\pi^2} \left(3 + \frac{4\tilde{m}_Q^2 - 6s}{3M^2} \right) \delta(s - \hat{E}_Q^2) \\
&\quad \left. \left. - \frac{x\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{24\pi^2} \delta(s - \hat{E}_Q^2) + \frac{(1-x)\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{8\pi^2} \delta(s - \hat{E}_Q^2) \right] \right\} e^{-\frac{s}{M^2}} + \frac{\langle \bar{q} q \rangle_{\rho_N}^2}{3} e^{-\frac{E_Q^2}{M^2}}, \tag{7}
\end{aligned}$$

$$\begin{aligned}
\frac{\lambda_{\Sigma_Q}^{*2} M_{\Sigma_Q}^*}{m_Q} e^{-\frac{E_Q^2}{M^2}} &= \int_{m_Q^2}^{s_0^*} ds \int_{x_i}^1 dx \left\{ \frac{3(1-x)^2(s - \hat{E}_Q^2)^2}{64\pi^4} - \frac{\langle q^\dagger i D_0 q \rangle_{\rho_N}}{3\pi^2} + \frac{2(1-x)\langle q^\dagger i D_0 q \rangle_{\rho_N}}{3\pi^2} \right. \\
&\quad \left[1 + 2s\delta(s - \hat{E}_Q^2) \right] - \frac{(1-x)^2 m_Q^2}{192\pi^2 x^3} \langle \frac{\alpha_s GG}{\pi} \rangle_{\rho_N} \delta(s - \hat{E}_Q^2) + \frac{1 - 2x^2}{64\pi^2 x^2} \langle \frac{\alpha_s GG}{\pi} \rangle_{\rho_N} \\
&\quad + \bar{E}_q \left[\frac{(1-x)\langle q^\dagger q \rangle_{\rho_N}}{2\pi^2} - \frac{\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{6\pi^2} \delta(s - \hat{E}_Q^2) + \frac{(1-x)\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{4\pi^2} \delta(s - \hat{E}_Q^2) \right. \\
&\quad - \frac{(1-x)\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{6\pi^2} \left(1 - \frac{s}{M^2} \right) \delta(s - \hat{E}_Q^2) - \frac{\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N}}{\pi^2} \delta(s - \hat{E}_Q^2) \\
&\quad + \frac{3(1-x)\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N}}{\pi^2} \delta(s - \hat{E}_Q^2) - \frac{2(1-x)\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N}}{\pi^2} \left(1 - \frac{s}{M^2} \right) \delta(s - \hat{E}_Q^2) \\
&\quad \left. \left. - \frac{(2x-1)\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{8\pi^2 x} \delta(s - \hat{E}_Q^2) \right] \right\} e^{-\frac{s}{M^2}} + \frac{2\langle \bar{q} q \rangle_{\rho_N}^2 + \langle q^\dagger q \rangle_{\rho_N}^2}{3} e^{-\frac{E_Q^2}{M^2}}, \tag{8}
\end{aligned}$$

$$\begin{aligned}
\lambda_{\Sigma_Q}^{*2} \Sigma_v e^{-\frac{E_Q^2}{M^2}} &= \int_{m_Q^2}^{s_0^*} ds \int_{x_i}^1 dx \left\{ \frac{x(1-x)(7s - 5\hat{E}_Q^2)\langle q^\dagger q \rangle_{\rho_N}}{4\pi^2} - \frac{x\langle q^\dagger g_s \sigma G q \rangle}{12\pi^2} \left[5 + 2\tilde{m}_Q^2\delta(s - \hat{E}_Q^2) \right] \right. \\
&\quad - \frac{x(1-x)\langle q^\dagger g_s \sigma G q \rangle_{\rho_N}}{2\pi^2} \left[-\frac{3}{4} + \left(\frac{\tilde{m}_Q^2}{6} - \frac{3s}{2} - \frac{s\tilde{m}_Q^2}{3M^2} \right) \delta(s - \hat{E}_Q^2) \right] \\
&\quad - \frac{x\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N}}{6\pi^2} \left[5 + 2\tilde{m}_Q^2\delta(s - \hat{E}_Q^2) \right] + \frac{x(1-x)\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N}}{\pi^2} \\
&\quad \left[\frac{5}{2} + \left(-\frac{5\tilde{m}_Q^2}{3} + 9s + \frac{2s\tilde{m}_Q^2}{M^2} \right) \delta(s - \hat{E}_Q^2) \right] + \frac{x\langle q^\dagger g_s \sigma G q \rangle}{8\pi^2} \left[\frac{1}{2} + \frac{\tilde{m}_Q^2}{3} \delta(s - \hat{E}_Q^2) \right] \\
&\quad + \frac{(1-x)\langle q^\dagger g_s \sigma G q \rangle}{16\pi^2} + \bar{E}_q \frac{4x(1-x)\langle q^\dagger i D_0 q \rangle_{\rho_N}}{3\pi^2} \left[4 + \tilde{m}_Q^2\delta(s - \hat{E}_Q^2) \right] \left\} e^{-\frac{s}{M^2}} \right. \\
&\quad \left. + \bar{E}_q \frac{2\langle q^\dagger q \rangle_{\rho_N}^2}{3} e^{-\frac{E_Q^2}{M^2}}, \tag{9}
\end{aligned}$$

where $\tilde{E}_Q^2 = \frac{m_Q^2}{x} + \bar{q}^2$, $E_Q^2 = m_Q^2 + \bar{q}^2$, $x_i = \frac{m_Q^2}{s}$, $s_0^* = \omega_0^2 = s_0 - \bar{q}^2$, the ω_0 is the threshold parameter, and $x_i \rightarrow 0$ in the spectral densities where the function $\delta(s - \hat{E}_Q^2)$ appears. We can obtain the masses $M_{\Sigma_Q}^*$, vector self-energies Σ_v and pole residues $\lambda_{\Sigma_Q}^*$ in the nuclear matter by solving the three equations with simultaneous iterations.

3 Numerical results and discussions

The input parameters of the QCD sum rules in the nuclear matter are taken as $\langle q^\dagger q \rangle_{\rho_N} = \frac{3}{2}\rho_N$, $\langle \bar{q}q \rangle_{\rho_N} = \langle \bar{q}q \rangle + \frac{\sigma_N}{m_u + m_d}\rho_N$, $\langle \bar{q}q \rangle = (-0.23 \text{ GeV})^3$, $m_u + m_d = 12 \text{ MeV}$, $\sigma_N = 45 \text{ MeV}$, $\langle \frac{\alpha_s GG}{\pi} \rangle_{\rho_N} = \langle \frac{\alpha_s GG}{\pi} \rangle - 0.65 \text{ GeV}\rho_N$, $\langle \frac{\alpha_s GG}{\pi} \rangle = (0.33 \text{ GeV})^4$, $\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N} + \frac{1}{12} \langle q^\dagger g_s \sigma G q \rangle_{\rho_N} = 0.031 \text{ GeV}^2 \rho_N$, $\langle \bar{q} i D_0 i D_0 q \rangle_{\rho_N} + \frac{1}{8} \langle \bar{q} g_s \sigma G q \rangle_{\rho_N} = 0.3 \text{ GeV}^2 \rho_N$, $\langle \bar{q} g_s \sigma G q \rangle_{\rho_N} = \langle \bar{q} g_s \sigma G q \rangle + 3.0 \text{ GeV}^2 \rho_N$, $\langle q^\dagger g_s \sigma G q \rangle_{\rho_N} = -0.33 \text{ GeV}^2 \rho_N$, $\langle q^\dagger i D_0 q \rangle_{\rho_N} = 0.18 \text{ GeV}\rho_N$, $\langle \bar{q} g_s \sigma G q \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = 0.8 \text{ GeV}^2$, $\langle \bar{q}q \rangle_{\rho_N}^2 = f \langle \bar{q}q \rangle_{\rho_N} \times \langle \bar{q}q \rangle_{\rho_N} + (1 - f) \langle \bar{q}q \rangle \times \langle \bar{q}q \rangle$, $f = 0.5 \pm 0.5$, $\bar{q}^2 = (0.27 \text{ GeV})^2$, and $\rho_N = (0.11 \text{ GeV})^3$ [3, 4].

We recover the QCD sum rules in the vacuum by taking the limit $\rho_N = 0$, then differentiate the Eqs.(7-8) with respect to $\frac{1}{M^2}$ respectively, and eliminate the pole residues λ_{Σ_Q} (here we smear the star * to denote the pole residues in the vacuum), then obtain two QCD sum rules for the masses M_{Σ_Q} , one can consult Ref.[13] for the technical details.

In the conventional QCD sum rules [1], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter M^2 and threshold parameter s_0 . We impose the two criteria on the heavy baryon states Σ_Q , and choose the threshold parameters $s_0 = (10.0 \pm 0.5) \text{ GeV}$ and $(43.5 \pm 0.5) \text{ GeV}$, the Borel parameters $M^2 = (1.9 - 2.7) \text{ GeV}$ and $(4.8 - 5.6) \text{ GeV}$ for the heavy baryons Σ_c and Σ_b , respectively [13]. Finally we obtain the values of the masses and pole residues $M_{\Sigma_c} = (2.54 \pm 0.15) \text{ GeV}$ and $(2.42 \pm 0.20) \text{ GeV}$, $M_{\Sigma_b} = (5.96 \pm 0.10) \text{ GeV}$ and $(5.73 \pm 0.16) \text{ GeV}$, $\lambda_{\Sigma_c} = (5.4 \pm 1.4) \times 10^{-2} \text{ GeV}^3$ and $(3.6 \pm 1.0) \times 10^{-2} \text{ GeV}^3$, $\lambda_{\Sigma_b} = (8.0 \pm 1.7) \times 10^{-2} \text{ GeV}^3$ and $(5.0 \pm 1.2) \times 10^{-2} \text{ GeV}^3$ from the QCD sum rules with respect to the spinor structures \not{q} and 1, respectively [13].

The experimental values of the masses are $M_{\Sigma_c^{++}} = (2454.03 \pm 0.18) \text{ MeV}$, $M_{\Sigma_c^+} = (2452.9 \pm 0.4) \text{ MeV}$, $M_{\Sigma_c^0} = (2453.76 \pm 0.18) \text{ MeV}$, $M_{\Sigma_b^+} = (5807.8 \pm 2.7) \text{ MeV}$ and $M_{\Sigma_b^-} = (5815.2 \pm 2.0) \text{ MeV}$ from the Particle Data Group [14], the average values are $M_{\Sigma_c} = 2.454 \text{ GeV}$ and $M_{\Sigma_b} = 5.812 \text{ GeV}$, respectively, which are consistent with the new experimental data from the CDF collaboration [15]. The predicted masses from both spinor structures \not{q} and 1 can reproduce the experimental data approximatively, however, the pole residues from the spinor structures \not{q} and 1 differ from each other greatly, i.e. the values have the hierarchy $\lambda_{\Sigma_Q}^{\not{q}} \gg \lambda_{\Sigma_Q}^1$, where the upper indexes denote the spinor structures. We can draw the conclusion that the two QCD sum rules in Eqs.(7-8) in the limit $\rho_N = 0$ can be satisfied by the approximate equal masses but unequal pole residues. If we obtain the masses M_{Σ_Q} by dividing Eq.(8) with Eq.(7) in the limit $\rho_N = 0$, the predictions $M_{\Sigma_c} = 1.40_{-0.05}^{+0.08} \text{ GeV}$ and $M_{\Sigma_b} = 3.56_{-0.10}^{+0.14} \text{ GeV}$ are much smaller than the experimental data due to the unequal pole residues. We can multiply the smaller masses by some coefficients to offset the effects of the unequal pole residues and reproduce the experimental data.

Take the same Borel parameters and threshold parameters as the QCD sum rules in the vacuum [13], we can obtain the hadronic parameters $M_{\Sigma_c}^* = 1.33_{-0.03}^{+0.06} \text{ GeV}$, $M_{\Sigma_b}^* = 3.33_{-0.07}^{+0.09} \text{ GeV}$, $\lambda_{\Sigma_c}^* = 2.46_{-0.16}^{+0.22} \times 10^{-2} \text{ GeV}^3$, $\lambda_{\Sigma_b}^* = 1.25_{-0.04}^{+0.08} \times 10^{-2} \text{ GeV}^3$, $\Sigma_v^{\Sigma_c} = 0.446_{-0.027}^{+0.035} \text{ GeV}$, $\Sigma_v^{\Sigma_b} = 0.776_{-0.035}^{+0.042} \text{ GeV}$ from the three coupled QCD sum rules in Eqs.(7-9). In the limit $\rho_N = 0$, we can obtain the values $M_{\Sigma_c} = 1.40_{-0.05}^{+0.08} \text{ GeV}$, $M_{\Sigma_b} = 3.56_{-0.10}^{+0.14} \text{ GeV}$, $\lambda_{\Sigma_c} = 1.99_{-0.26}^{+0.29} \times 10^{-2} \text{ GeV}^3$, $\lambda_{\Sigma_b} = 8.73_{-0.65}^{+0.90} \times 10^{-3} \text{ GeV}^3$. In calculations, we have taken the assumption that the pole residues $\lambda_{\Sigma_Q}^*$ in the QCD sum rules (see Eqs.(7-9)) have the same values. In fact, those QCD sum rules can be satisfied with approximately the same in-medium masses $M_{\Sigma_Q}^*$ and vector self-energies $\Sigma_v^{\Sigma_Q}$, but different pole residues $\lambda_{\Sigma_Q}^*$, which can be denoted as $\lambda_{\Sigma_Q}^*(1)$, $\lambda_{\Sigma_Q}^*(2)$ and $\lambda_{\Sigma_Q}^*(3)$ from the

QCD sum rules in Eqs.(7-9) respectively. We draw this conclusion tentatively based on the QCD sum rules for the heavy baryon states in the vacuum [13], and normalize the masses from the QCD sum rules in the vacuum to the experimental data to study the mass modifications in the nuclear matter,

$$\delta M_{\Sigma_Q} = \frac{M_{\Sigma_Q}^* - M_{\Sigma_Q}}{M_{\Sigma_Q}} \times M_{\Sigma_Q}^{\text{exp}}, \quad (10)$$

and obtain the central values $\delta M_{\Sigma_c} = -123 \text{ MeV}$ and $\delta M_{\Sigma_b} = -375 \text{ MeV}$, the δM_{Σ_Q} are the scalar self-energies $\Sigma_s^{\Sigma_Q}$. The Σ_Q baryons have a heavy quark besides two light quarks, the heavy quark interacts with the nuclear matter through the exchange of the intermediate gluons, the contributions from the gluon condensates are of minor importance and the modifications of the gluon condensates in the nuclear matter are mild, we expect the ratios of the mass-shifts $\delta M_{\Sigma_Q}/M_{\Sigma_Q}$ are smaller than that of the nucleons. From Fig.1, we can obtain the mass differences $\Delta M_{\Sigma_Q} = M_{\Sigma_Q}^* - M_{\Sigma_Q}$, $\Delta M_{\Sigma_c} = -0.07 \pm 0.02 \text{ GeV}$, $\Delta M_{\Sigma_b} = -0.24 \pm 0.04 \text{ GeV}$, the uncertainties of the mass differences originate from the Borel parameters are about 29% and 17%, respectively. The ratios are $\frac{\Delta M_{\Sigma_c}}{M_{\Sigma_c}} = -5.0\%$ and $\frac{\Delta M_{\Sigma_b}}{M_{\Sigma_b}} = -6.7\%$, the mass modifications are rather small.

If we take into account the uncertainties of the heavy quark masses and threshold parameters, $\delta m_c = \pm 0.1 \text{ GeV}$, $\delta m_b = \pm 0.1 \text{ GeV}$, $\delta s_{\Sigma_c}^0 = \pm 0.5 \text{ GeV}^2$, $\delta s_{\Sigma_b}^0 = \pm 0.5 \text{ GeV}^2$ [13], the values of the mass differences ΔM_{Σ_Q} survive approximately, no additional uncertainties are introduced. At the interval $f = 0 \sim 1$, the masses $M_{\Sigma_Q}^*$ decrease monotonously with the increase of the parameter f , the uncertainty $\delta f = \pm 0.5$ leads to the uncertainties $\pm 0.06 \text{ GeV}$ and $\pm 0.21 \text{ GeV}$ for the masses $M_{\Sigma_c}^*$ and $M_{\Sigma_b}^*$ respectively in the nuclear matter, and the corresponding uncertainties for the mass-shifts δM_{Σ_c} and δM_{Σ_b} are $\pm 105 \text{ MeV}$ and $\pm 343 \text{ MeV}$, respectively.

If we take the Ioffe current to interpolate the proton, the QCD sum rules indicate that there exists a positive vector self-energy $\Sigma_v^N = (0.23 - 0.35) \text{ GeV}$ with the typical values of the relevant condensates and other input parameters, which is consistent with the values of the vector self-energies in the relativistic nuclear physics phenomenology, on the other hand, although the scalar self-energy depends strongly on the in-medium four-quark condensate and the nucleon σ term, a reasonable negative scalar self-energy can be obtained with the suitable parameters [3]. There exists significant cancelation between the scalar and vector self-energies, which leads to a quasinucleon energy close to the free-space nucleon mass and satisfies the empirical expectation that the quasinucleon energy is shifted only slightly in nuclear matter relative to the free-space mass. The in-medium self-energies Σ_s^N and Σ_v^N can be written as

$$\begin{aligned} \Sigma_s^N &= -\frac{8\pi^2 \sigma_N \rho_N}{M^2(m_u + m_d)}, \\ \Sigma_v^N &= \frac{32\pi^2 \rho_N}{M^2}, \end{aligned} \quad (11)$$

in the leading order approximation, $\Sigma_s^N/\Sigma_v^N \approx -1$, which indicates a substantial cancelation between self-energies Σ_s^N and Σ_v^N in the nuclear matter. The mean-field models predicate that the typical self-energies of the nucleons in nuclear matter saturation density are $\Sigma_s^N \approx -350 \text{ MeV}$ and $\Sigma_v^N \approx +300 \text{ MeV}$ respectively, which correspond to the real energy-independent optical potentials S and V , and significant cancelation between the potentials occurs, the effective non-relativistic central potential is about tens of MeV. If the same mechanism works for the Σ_c baryon, the vector self-energy should be $\Sigma_v^{\Sigma_c} \approx +123 \text{ MeV}$ rather than $+446 \text{ MeV}$ according to the unequal pole residues $\lambda_{\Sigma_Q}^*(1) \neq \lambda_{\Sigma_Q}^*(2) \neq \lambda_{\Sigma_Q}^*(3)$, the total self-energy $\Sigma_s^{\Sigma_c} + \Sigma_v^{\Sigma_c} \approx 0$ under the condition $\lambda_{\Sigma_Q}^*(3)/\lambda_{\Sigma_Q}^*(2) \approx 1.9$, then the quasi- Σ_c energy in the nuclear matter close to the free-space Σ_c mass. The present prediction of the mass-shift $\delta M_{\Sigma_c} = -123 \text{ MeV}$ can be confronted with the experimental data from the CBM and PANDA collaborations in the future [8, 9], where the properties of the charmed baryons in the nuclear matter will be studied.

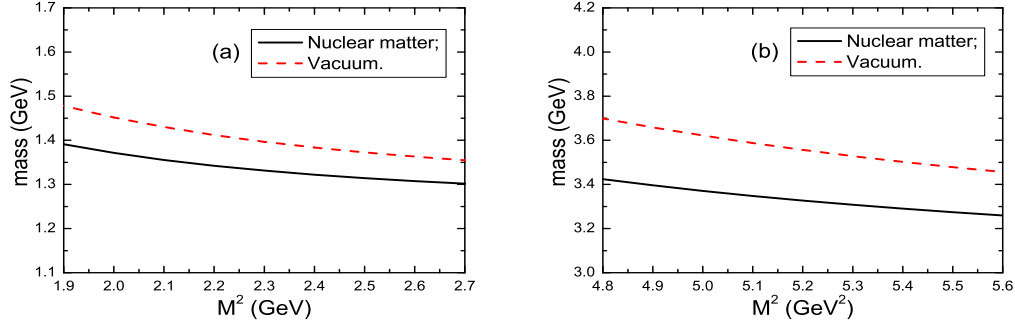


Figure 1: The masses from the QCD sum rules in the vacuum and in the nuclear matter versus the Borel parameter M^2 , the (a) and (b) denote the Σ_c and Σ_b baryons, respectively.

In the non-relativistic harmonic-oscillator potential model, the spectrum of the bound states (the energies E_n and the wave-functions $\psi_n(x)$) and the exact correlation functions are known precisely. In Ref.[16], Lucha, Melikhov and Simula try to fit the effective threshold parameter so as to reproduce both the ground energy E_0 and the pole residue R_0 ($\psi_0^*(0)\psi_0(0)$), or reproduce the ground energy E_0 only and take the pole residue R_0 as a calculated parameter. They observe that the pole residue R_0 is determined to a great extent by the continuum contributions, and draw the conclusion that the ground-state parameters extracted from the QCD sum rules have uncontrolled systematic errors if the continuum contributions are not known exactly and modeled by an effective continuum. In the real QCD world, the hadronic spectral densities are not known exactly, the pole residues or decay constants in some (or most) cases are not experimentally measurable quantities, and should be calculated by some theoretical approaches, the true values are difficult to obtain. On the other hand, the hadronic spectrum densities in the non-relativistic harmonic-oscillator potential model are of the Dirac δ function type even in the limit $n \rightarrow \infty$, while in the case of the QCD, the widths of the higher radial excited states become broader gradually before submerging into the asymptotic quarks and gluons. For example, the widths of the π , $\pi(1300)$, $\pi(1800)$, \dots are ~ 0 GeV, $(0.2 - 0.6)$ GeV, 0.208 ± 0.012 GeV, \dots respectively, and the widths of the K , $K(1460)$, $K(1830)$, \dots are ~ 0 GeV, $\sim (0.25 - 0.26)$ GeV, ~ 0.25 GeV, \dots respectively [14]. We cannot estimate the unknown systematic uncertainties of the QCD sum rules before the spectral densities in both the QCD and phenomenological sides are known with great accuracy. In the present case, the situation is even worse, the theoretical works on the Σ_Q baryons in the nuclear matter are rare, and no experimental data exist.

4 Conclusion

In this article, we extend our previous work on the Λ_Q type heavy baryons Λ_Q to study the properties of the Σ -type heavy baryons Σ_Q in the nuclear matter using the QCD sum rules, and derive three coupled QCD sum rules for the masses $M_{\Sigma_Q}^*$, vector self-energies Σ_v and pole residues $\lambda_{\Sigma_Q}^*$ in the nuclear matter, then obtain the values $M_{\Sigma_c}^* = 1.33_{-0.03}^{+0.06}$ GeV, $M_{\Sigma_b}^* = 3.33_{-0.07}^{+0.09}$ GeV, $\lambda_{\Sigma_c}^* = 2.46_{-0.16}^{+0.22} \times 10^{-2}$ GeV³, $\lambda_{\Sigma_b}^* = 1.25_{-0.04}^{+0.08} \times 10^{-2}$ GeV³, $\Sigma_v^{\Sigma_c} = 0.446_{-0.027}^{+0.035}$ GeV, $\Sigma_v^{\Sigma_b} = 0.776_{-0.035}^{+0.042}$ GeV. In the limit $\rho_N = 0$, the predictions $M_{\Sigma_c} = 1.40_{-0.05}^{+0.08}$ GeV and $M_{\Sigma_b} = 3.56_{-0.10}^{+0.14}$ GeV are much smaller than the experimental data due to the unequal pole residues from different spinor structures $\gamma \cdot q$ and 1. We normalize the masses $M_{\Sigma_c} = 1.40$ GeV and $M_{\Sigma_b} = 3.56$ GeV from the QCD sum rules in the vacuum to the experimental data, and obtain the mass-shifts in the nuclear matter $\delta M_{\Sigma_c} = -123$ MeV and $\delta M_{\Sigma_b} = -375$ MeV, which can be confronted with the experimental

data in the future.

Acknowledgments

This work is supported by National Natural Science Foundation, Grant Number 11075053, and the Fundamental Research Funds for the Central Universities.

References

- [1] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385; L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1.
- [2] E. G. Drukarev and E. M. Levin, Prog. Part. Nucl. Phys. **27** (1991) 77; E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, Prog. Part. Nucl. Phys. **47** (2001) 73; E. G. Drukarev, Prog. Part. Nucl. Phys. **50** (2003) 659.
- [3] T. D. Cohen, R. J. Furnstahl, D. K. Griegel and X. M. Jin, Prog. Part. Nucl. Phys. **35** (1995) 221.
- [4] X. M. Jin, T. D. Cohen, R. J. Furnstahl and D. K. Griegel, Phys. Rev. **C47** (1993) 2882; X. M. Jin, M. Nielsen, T. D. Cohen, R. J. Furnstahl and D. K. Griegel, Phys. Rev. **C49** (1994) 464.
- [5] A. Hayashigaki, Prog. Theor. Phys. **101** (1999) 923; F. Klingl, S. Kim, S. H. Lee, P. Morath and W. Weise, Phys. Rev. Lett. **82** (1999) 3396; S. Kim and S. H. Lee, Nucl. Phys. **A679** (2001) 517; C. M. Ko and S. H. Lee, Phys. Rev. **C67** (2003) 038202; K. Morita and S. H. Lee, Phys. Rev. Lett. **100** (2008) 022301; K. Morita and S. H. Lee, Phys. Rev. **C77** (2008) 064904; A. Kumar and A. Mishra, Phys. Rev. **C82** (2010) 045207.
- [6] A. Hayashigaki, Phys. Lett. **B487** (2000) 96; T. Hilger, R. Thomas and B. Kampfer, Phys. Rev. **C79** (2009) 025202; T. Hilger and B. Kampfer, Nucl. Phys. Proc. Suppl. **207-208** (2010) 277; Z. G. Wang and T. Huang, Phys. Rev. **C84** (2011) 048201; T. Hilger, B. Kampfer and S. Leupold, Phys. Rev. **C84** (2011) 045202; S. Zschocke, T. Hilger and B. Kampfer, Eur. Phys. J. **A47** (2011) 151.
- [7] Z. G. Wang, Eur. Phys. J. **C71** (2011) 1816.
- [8] B. Friman et al, "The CBM Physics Book: Compressed Baryonic Matter in Laboratory Experiments", Springer Heidelberg.
- [9] M. F. M. Lutz et al, arXiv:0903.3905.
- [10] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. **D12** (1975) 147; R. L. Jaffe, hep-ph/0001123.
- [11] Z. G. Wang, Eur. Phys. J. **C71** (2011) 1524; Z. G. Wang, arXiv:1112.5910.
- [12] J. G. Koerner, D. Pirjol and M. Kraemer, Prog. Part. Nucl. Phys. **33** (1994) 787; F. Hussain, G. Thompson and J. G. Koerner, arXiv:hep-ph/9311309.
- [13] Z. G. Wang, Eur. Phys. J. **A44** (2010) 105.
- [14] K. Nakamura et al, J. Phys. **G37** (2010) 075021.
- [15] T. Aaltonen et al, Phys. Rev. **D84** (2011) 012003; T. Aaltonen et al, arXiv:1112.2808.
- [16] W. Lucha, D. Melikhov and S. Simula, Phys. Rev. **D76** (2007) 036002; W. Lucha, D. Melikhov and S. Simula, Phys. Lett. **B657** (2007) 148.